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On the generalised Glauber kinetics for a one-dimensional Potts chain

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Abstract. The one-dimensional Potts chain dynamics is analysed by means of the direct space renormalisation technique. In contrast to previous work we need no restriction on the value of the Potts dimensionality q. We use the generalised Glauber kinetics for the chain and find that the value of the dynamical critical exponent z is not universal, i.e. it depends on both Potts dimensionality and the transition rates asymmetry coefficient λ.

The Potts model (Potts 1952, Wu 1982)—a generalisation of the Ising model—has again become the subject of intensive research (Forgacs *et al* 1980, Lage 1985, Weir and Kosterlitz 1986, Weir *et al* 1986, Sherrrington 1986). While static properties of the Potts model have been extensively studied and are pretty well understood (see Wu 1982, Sherrington 1986), the dynamical behaviour of that model is rather poorly known. An understanding of the dynamics of the Potts model is of great importance in studies of various dynamical phenomena, such as the Snoek effect (Dattagupta *et al* 1982), liquid-glass transition (Kree *et al* 1987) and Potts glass theory (Kirkpatrick and Wolynes 1987, Carmesin and Binder 1988). Similar to the Ising model, the Potts model does not possess 'intrinsic' dynamics. Thus in the analysis of its kinetics we have to use the same method as used in the Ising case. The simplest way is to take the Glauber model (Glauber 1963) formulated for Ising systems (i.e. Potts model with $q=2$) and generalise the model for arbitrary q.

The crucial point in the Glauber analysis is the construction of the master equation and therefore the choice of the transition rates in the master operator. Those transition rates have to obey certain conditions, out of which the detailed balance is the most important one. In the Ising case any choice of the transition rates fulfilling the detailed balance leads to the same values of dynamical exponent $z=2$. For $q>2$ one obtains various values of z, depending on the definition of the transition rates. Different choices of the transition rates were the main reasons for various values of z calculated by Lage (1985), Weir and Kosterlitz (1986) and Weir *et al* (1986).

Weir and Kosterlitz (1986) have calculated the exponent z by noting that the $q=2^n$ Potts model can be mapped on n coupled Ising models with 2^n spin interactions. They have used the real-space renormalisation group approach introduced by Achiam (1983

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and references therein) for dynamical problems. In this paper we shall use the same method, but without any restriction on the value of Potts dimensionality q or special choice of the transition rates. For $q=2^n$ our results agree with those of Weir and Kosterlitz (1986). For the general case our results differ from those of Weir *et al* (1986).

The 1D Potts system can be described by the Hamiltonian:

$$\beta H = -K \sum_{ij} S_i \cdot S_j \tag{1}$$

where the Potts spin on site i assumes the values: $s_\alpha = 0, 1, 2, \ldots, q-1$ and $S_\alpha \cdot S_\beta = q\delta_{\alpha\beta} - 1$. The summation in (1) is restricted to nearest neighbours.

The Glauber-like master equation which governs this model dynamics is

$$-\tau_0 \frac{\partial}{\partial t} P(S, t) = \hat{L}P(S, t) = \sum_i \sum_{\tilde{S}_i \neq S_i} [1 - P(S_i \to \tilde{S}_i)] W(S_i \to \tilde{S}_i) P(S, t) \tag{2}$$

where $P(S_i \to \tilde{S}_i)$ is the spin-flip operator that changes the state of the vector S at the lattice site i:

$$P(S_i \to \tilde{S}_i) P(S_1 \ldots S_i \ldots S_N, t) = P(S_1 \ldots \tilde{S}_i \ldots S_N, t). \tag{3}$$

The transition rates $W(S_i \to \tilde{S}_i)$ should satisfy the detailed balance condition:

$$(1 - P(S_i \to \tilde{S}_i)) W(S_i \to \tilde{S}_i) P_e(S) = 0 \tag{4}$$

where $P_e(S)$ describes the equilibrium state:

$$P_e = \frac{\exp(-\beta H)}{\mathrm{Tr}[\exp(-\beta H)]}.$$

The very broad class of transition rates $W(S_i \to \tilde{S}_i)$ which satisfy (4), for the Hamiltonian (1), has the form:

$$W(S_i \to \tilde{S}_i) = C(S_1, \ldots, S_{i-1}, S_{i+1}, \ldots, S_N) \frac{1}{P_e} (P_e P(S_i \to \tilde{S}_i) P_e)^\lambda \tag{5}$$

where the real parameter $\lambda \in [0, 1]$. The function C is a *weak* function (Haake and Thol 1980, Achiam 1983) and thus it does not influence the value of the exponent z. Hence we can set it equal to one.

In order to understand the meaning of the parameter λ in a better way we rewrite $W(S_i \to \tilde{S}_i)$ as

$$W(S_i \to \tilde{S}_i) \propto \exp\{K[\tfrac{1}{2}(S_{i-1}\tilde{S}_i + S_{i+1}\tilde{S}_i - S_{i-1}S_i - S_{i+1}S_i) + (\lambda - \tfrac{1}{2})(S_{i-1}\tilde{S}_i + S_{i+1}\tilde{S}_i + S_{i-1}S_i + S_{i+1}S_i)]\}. \tag{6}$$

For $\lambda = \frac{1}{2}$ this became identical to the expression used by Weir and Kosterlitz (1986). Such a definition seems the most natural when we have no additional information about the internal dynamics mechanism. An arbitrary coefficient λ multiplying the sum of the energy of initial and final states causes the transition rates to depend not only on the differences between energies of states but also on the values of those energies. Now, the transition between two states can be 'faster' when these states are close to each other in energy and 'slower' when they are far apart. For $\lambda = 0$ the transition rate is independent of the energy of the final state while in the case $\lambda = 1$ it is independent of the energy of the initial state. As we shall see, the critical exponent z depends in a crucial way on the value of that rate asymmetry coefficient. Formally

we can also consider the generalised rates W given by (5) with negative λ as well as with $\lambda > 1$. We have no clear insight into the physical meaning of the negative λ. Thus we shall restrict our analysis to positive values of the rate asymmetry coefficient. For the sake of completeness, however, we will also quote results obtained in the negative λ sector of the parameter space. Note also that in the Ising case $\tilde{S}_i = -S_i$ and the λ term vanishes. The Ising vector has no choice when its state changes. The rate asymmetry coefficient plays no role and that is why the exponent z is universal.

From now on we follow the usual Achiam procedure (Achiam 1983). The master equation (2) can now be rewritten as

$$-\tau_0 P_e \frac{\partial}{\partial t}\phi(S, t) = \hat{L}\phi(S, t) = \sum_i \sum_{\tilde{S}_i \neq S_i} P_e(S) W(S_i \to \tilde{S}_i)(1 - P(S_i \to \tilde{S}_i))\phi(S, t) \tag{7}$$

where

$$\phi(S, t) = \frac{P(S, t)}{P_e(S)}$$

measures the deviation of the system from the equilibrium state.

Since we shall consider linear deviations from the equilibrium state only, we write

$$\phi(S, t) = 1 + h(t) \sum_i S_i. \tag{8}$$

Following Achiam (1983) we apply the renormalisation transformation

$$T(S', S) = \prod_i \delta(S'_i - S_{2i}) \tag{9}$$

to both sides of the master equation (7) and to the transfer matrix:

$$M_{n,n+1}(K) = \exp(KS_n \cdot S_{n+1}). \tag{10}$$

This last operation results in

$$A(K) M'_{n,n+1}(K') = M_{2n,2n+1}(K) M_{2n+1,2n+2}(K) \tag{11}$$

where $A(K)$ is the usual renormalisation constant.

Equation (11) gives the recursion relations, for $q \geq 2$,

$$\begin{aligned} x^{2(q-1)} + (q-1)x^{-2} &= A(x')^{q-1} \\ 2x^{q-2} + (q-2)x^{-2} &= A(x')^{-1} \end{aligned} \tag{12}$$

where $x = \exp(K)$.

The only fixed point of (12) is that at zero temperature, i.e. $x = \infty$.

Now the transformation of the master equation (7) simplifies when one rewrites the equilibrium distribution function as $P_e(S) = \prod_n M_{n,n+1}$. The left-hand side of the master equation, near the fixed point, assumes the form:

$$\sum_S T(\tilde{S}, S) P_e \phi = A^{N/2} P'_e \left(1 + h' \sum_i S'_i\right) \tag{13}$$

where the rescaled $h' = 2h$.

Then the RHS of the master equation (7) can be rewritten as

$$\begin{aligned} \sum_S TL\phi = \sum_n \sum_{\tilde{S}_{2n} \neq S_{2n}} h(S_{2n} - \tilde{S}_{2n}) A^{N/2} \prod_{i \neq n-1, n} M'_{i,i+1}(M_{2n-1,2n-1} \tilde{M}_{2n-1,2n}) \\ \times (\tilde{M}_{2n,2n+1} M_{2n+1,2n+2}) \end{aligned} \tag{14}$$

where

$$\tilde{M}_{2n-1,2n} = M_{2n-1,2n} \exp[\lambda K S_{2n-1}\tilde{S}_{2n} - (1-\lambda)KS_{2n-1}S_{2n}] \tag{15}$$

and we have used the detailed balance condition.

The renormalisation relation for $\tilde{M}$ has the form

$$M\tilde{M} = \kappa A\tilde{M}'. \tag{16}$$

Near the fixed point κ can be calculated from the relation

$$\kappa A x^{-2\lambda} \simeq x^{q-1-2\lambda}. \tag{17}$$

For q not smaller than 2 we have

$$\kappa = (\tfrac{1}{2})^{[q-1-\lambda(q-2)]/q}. \tag{18}$$

Writing the renormalised form of the master equation (7) as

$$-\tau_0 P_e' \frac{\partial}{\partial t}\phi'(S', t) = \tfrac{1}{2}\kappa^2 L'\phi'(S', t) \tag{19}$$

and defining the renormalised timescale as

$$\tau_0' = \tau_0 2\kappa^{-2} = 2^{\kappa}\tau_0 \tag{20}$$

we obtain the following expression for the critical exponent z:

$$z = 3 - 2\lambda + 2\frac{2\lambda - 1}{q}. \tag{21}$$

For the sake of completeness we also quote here the similar result obtained for negative λ:

$$z = 3 + 2\frac{2\lambda - 1}{q}. \tag{22}$$

Equation (21) constitutes our main result, which we can now compare with those from Weir and Kosterlitz (1986) and Lage (1985).

First, notice that, for the Ising case, i.e. $q = 2$, we have $z = 2$ independent of λ. For the arbitrary q but $\lambda = \frac{1}{2}$ case discussed by Weir and Kosterlitz (1986) we again have $z = 2$. In the general case our critical exponent z depends on both λ and the Potts dimensionality q. This should be contrasted with the Weir *et al* (1986) form where z depends on $\mu = 1 - \lambda$ only. Also note that the Lage result $z = 3$ can only be obtained in the limit $\lambda \to 0$ for fixed q and then letting $q \to \infty$. In figure 1 we have shown the Potts dimensionality q dependence of the exponent z for various values of the rate asymmetry coefficient. The shaded region, symmetric with respect to the $z = 2$ line, is bounded by $z = 3 - 2/q$ and $z = 1 + 2/q$ lines, corresponding to the $\lambda = 0$ and $\lambda = 1$ cases, respectively.

In conclusion we have shown that the standard application of the dynamical renormalisation group procedure proposed by Achiam (1983) to the one-dimensional Potts chain shows that the critical exponent z for that model is not universal, i.e. it depends on the explicit form of the transition rate used in the construction of the Glauber master operator (7). We were able to derive the explicit form of the exponent z for arbitrary value of the Potts dimensionality q and therefore our results generalise previous analyses restricted to $q = 2^n$. The spin-flip asymmetry coefficient λ contains additional information about the internal dynamical property of the Potts model,

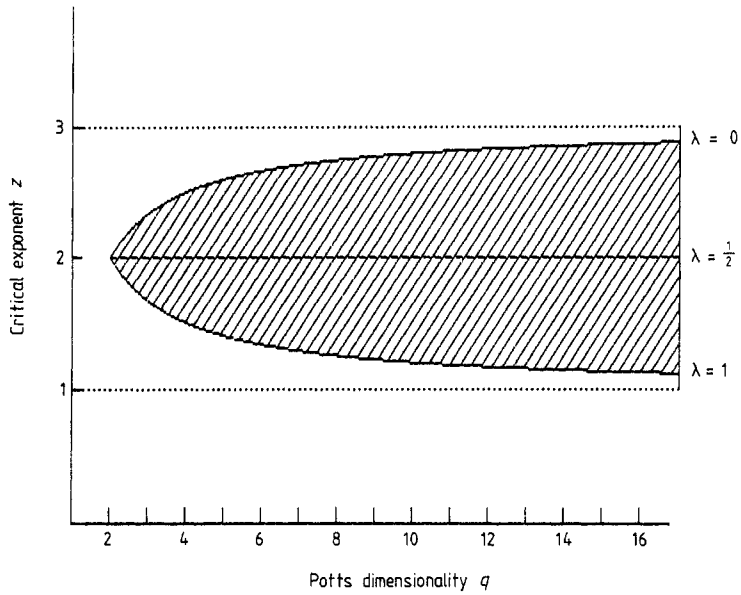

Figure 1. The value of the dynamical critical exponent z plotted as a function of the Potts dimensionality q for various values of the rate asymmetry coefficient λ. z assumes any value within the shaded region bounded by the curves corresponding to the extreme values of $\lambda = 0, 1$ (full lines). The broken line shows the value of z for $\lambda = \frac{1}{2}$, which is independent of q.

namely it measures the relative 'speed' of spin-flip, depending on the distance between the state energies. For most of the applications that parameter is positive and equal to $\frac{1}{2}$.

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